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Phase Report
May 1977

DESIGN OF TERRESTRIAL-SATELLITE HYBRID COMMUNICATION NETWORKS

Purdue University

ROME AIR DEVELOPMENT CENTER
Air Force Systems Command
Griffiss Air Force Base, New York 13441

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In a hybrid communication system, comprising both satellite and terrestrial networks, part of the network topology is adaptable. The availability of adaptable topology and alternate routing in such networks provides the designer with considerable freedom to insure satisfactory network and point-to-point grades of service.

The complexity inherent in such a design problem is discussed. A heuristic

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algorithm--which can be implemented on a digital computer--for designing hybrid networks with the aim of reducing all node-to-node grades of service below a prescribed value is given. Several topics of further research are presented.

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PREFACE

This effort was conducted by Purdue University under the sponsorship of the Rome Air Development Center Post-Doctoral Program.

The RADC Post-Doctoral Program is a cooperative venture between RADC and some sixty-five universities eligible to participate in the program. Syracuse University (Department of Electrical Engineering), Purdue University (School of Electrical Engineering), Georgia Institute of Technology (School of Electrical Engineering), and State University of New York at Buffalo (Department of Electrical Engineering) act as prime contractor schools with other schools participating via sub-contracts with the prime schools. The U.S. Air Force Academy (Department of Electrical Engineering), Air Force Institute of Technology (Department of Electrical Engineering), and the Naval Post Graduate School (Department of Electrical Engineering) also participate in the program.

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Center (ADTC), Air Force Communications Service (AFCS), Aerospace Defense Command (ADC), Hq USAF, Defense Communications Agency (DCA), Navy, Army, Aerospace Medical Division (AMD), and Federal Aviation Administration (FAA).

Further information about the RADC Post-Doctoral Program can be obtained from Mr. Jacob Scherer, RADC/RBC, Griffiss AFB, NY, 13441, telephone Autovon 587-2543, commercial (315) 330-2543.

1. INTRODUCTION

A considerable amount of effort has previously been devoted to optimizing the performance of terrestrial communication systems [1,2]. The objective of such design efforts has been to obtain procedures for optimizing the network performance with regard to cost, consumer satisfaction, etc.

With the growing availability of communication satellites, new optimization procedures for the hybrid systems, comprising the satellite and terrestrial communication networks, must be considered. With the growing traffic demands--and the poor performance of some of the existing terrestrial system, e.g., European AUTOVON--new algorithms for improving network performance must be sought.

The hybrid system, like the terrestrial systems, could be used in a fixed network configuration; that is, the satellite capacity available to different ground stations is fixed and inflexible. Such an operation would, however, ignore the most important feature associated with satellite communications: network adaptability. The network adaptability property coupled with adaptive routing procedures provides extra flexibility to the designer to improve the performance of the hybrid system.

One aspect of communication network performance optimization deals with the routing of calls. The improvement of network performance for the terrestrial systems through routing table updating, as previously investigated [3], made tacit use of the fact that the network configuration is fixed.

With the satellite system the network topology is no longer fixed. Satellite circuits can be allocated depending on the demands from the ground stations. The up-link and down-link transmission delay prohibits any alternate (multiple hop) routing on the satellite system.

Thus, in the terrestrial system the network configuration is fixed, but alternate routing is possible. In the satellite system, the network topology is variable but only direct routing is allowed. In the hybrid system, therefore, the need for proper use of these available features is evident.

This report documents a preliminary investigation of the routing problem in hybrid communication networks. A search of the literature reveals no existing work on such networks.

Section 2 deals with the modelling and the analysis of the hybrid communication networks.

In section 3, the optimization problem for such networks is formulated.

Section 4 addresses some of the basic questions regarding the effect of network configuration and alternate routing on trunk group sizing. A solution to the problem posed in section 3 is also presented in this section.

Finally, several topics of future research are given in section 5.

2. MODELING, ROUTING STRATEGIES AND ANALYSIS OF HYBRID COMMUNICATION NETWORKS

2.1 Preliminary Definitions

A communication network can be modeled by a graph, where each node in the graph represents a switching center (exchange), and each link represents a trunk group. The capacity of each link is defined in terms of the number of trunks in the link. Each trunk represents one message channel in the link.

A network with n nodes has n x (n-1) distinct node pairs. The traffic demands for all node pairs can be described by a square matrix of order n, called the traffic matrix. The diagonal entries of this matrix are vacant (have no physical meaning). Each off-diagonal element has two nodes associated with it: the source (S) node and the destination (D) node. The source node is where the call originate; and the destination node is where the calls terminate (or are destined to). The elements of the traffic matrix may be in units of Erlangs or hundred-call seconds (abbreviated CCS), one CCS being equal to 1/36th of an Erlang.

The class of communication networks dealt with in this report belong to the circuit-switched type. In circuit-switched networks, a path is set up between the source and the destination pair prior to the starting of message transmission.[1]

The nodes are assumed to be perfectly reliable (blocking probability = 0). The links, however, are modeled on the basis of their capacity, and their availability is, therefore, assumed to be probabilistic. Based on this model, the link grade of service is defined as the calls blocked by a link divided by the calls attempted on the link.

[1] For example, in the SPADE system, the path (channel) is set up prior to message transmission.

Due to the probabilistic nature of the availability of links, the actual calls completed between a source-to-destination pair will be less than the originating calls. To this end, we define the node-to-node grade of service (abbreviated NNGOS) as the lost load between a source→destination pair divided by the total load destined between the source→destination pair. Two other quantities that are often used in describing the network performance are: (1) the network grade of service is defined as the total traffic lost in the network divided by the total traffic originating in the network, and (2) the node grade of service[2] is defined as the total traffic lost at a node divided by the total traffic originating at the node.

2.2 Routing Strategies

The routing strategy employed in a network can be completely described by a routing table and a call control rule.

The routing table for an n-node network can be expressed by a matrix of order n. This matrix is used to dictate the "preference scheme" for routing calls. The order of the entries in the routing table indicates the preference scheme.

The need to define a call control rule arises due to ambiguities that may arise in the routing table specification [3]. A number of call control rules exist in various communication networks [4]. In this report, we shall deal with the originating office control rule. The originating office control (abbreviated OOC) rule may be stated as follows:

---

[2]Note that this quantity is due to the probabilistic nature of the availability of the links and not the nodes (as assumed earlier).

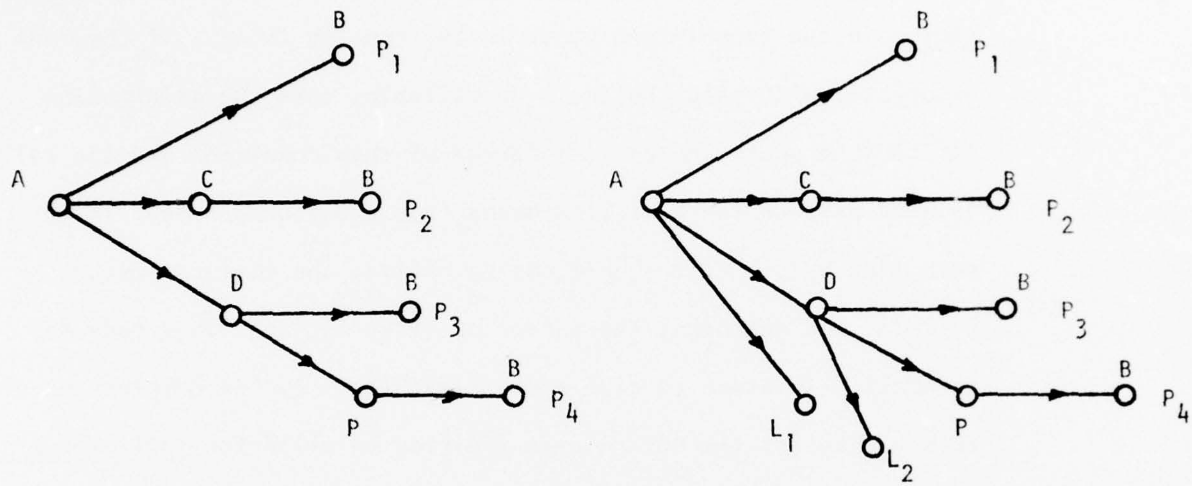
"If node I is the originating node for a call destined to node D, then the call is routed to some adjacent node specified by the from-I-to-D block in the routing table. If I is a tandem node, then a call reaching node I is routed only to the node appearing as the first choice in the from-I-to-D block in the routing table. If the link associated with this choice is unavailable, then the originating office (the source node) is informed of this condition and the call is attempted on the next link going from the source node. If no such link exists at the originating office, the call is lost."

In some networks, the number of links incident to a node may be small. In order to give added flexibility to the originating office to route calls, the OOC rule is modified to allow for spill forward action. According to the originating office control with spill forward rule, the originating office may pass (spill) the control of a call to a tandem switch, which then assumes the role of the originating office. In general, the number of paths available to route calls increases with the spill forward action.

The process of routing can be depicted with the aid of a route tree, which essentially shows all the paths defined by the routing table and the call control rule. In order to completely define the routing procedure, the concept of augmented route tree is used [3]. The augmented route tree can be obtained from a route tree in the following manner:

Add one directed branch below all the branches leaving an originating node or the spill node in the route tree, and label this terminal node as L_i.

The route tree of Fig. 2.1(a) depicts the routing of calls from node A to node B. The augmented route tree of Fig. 2.1(b) depicts the routing as well as the loss of calls for the same node pair.



(a) Route Tree (b) Augmented Route Tree

Figure 2.1

2.3 Hybrid Communication System Network Structure

The definitions introduced in section 2.1 are sufficient to model a terrestrial communication systems. The same modelling scheme can be used to represent a hybrid communication system. However, if a demand assignment scheme is to be implemented on a digital computer, the need to distinguish the satellite portion of the hybrid system is evident. In this section, we will present a network model that can be used to represent any hybrid communication system.

In the terrestrial communication system, the nodes represent a switching center of the network, and a link is used to connect two switching centers. In such networks, the link capacity and the location of the switching centers are fixed. Thus, the network has a fixed configuration.

In the satellite communication system, the nodes represent a transit center (TC) or a ground station (GS). Due to the large expense involved in obtaining and maintaining a ground station, not all the countries have such facilities. Such countries can access the satellite by routing the call to the nearest ground station. Thus the links in such networks represent connections between two transit centers, two ground stations, or a transit center and a ground station. The capacities of the links connecting two transit centers or transit center and ground station are, in general, fixed. The capacity of the link whose both ends terminate at a ground station are variable.

The hybrid communication system, which is a combination of the terrestrial and the satellite system, can be represented by the block diagram of Fig. 2.2.

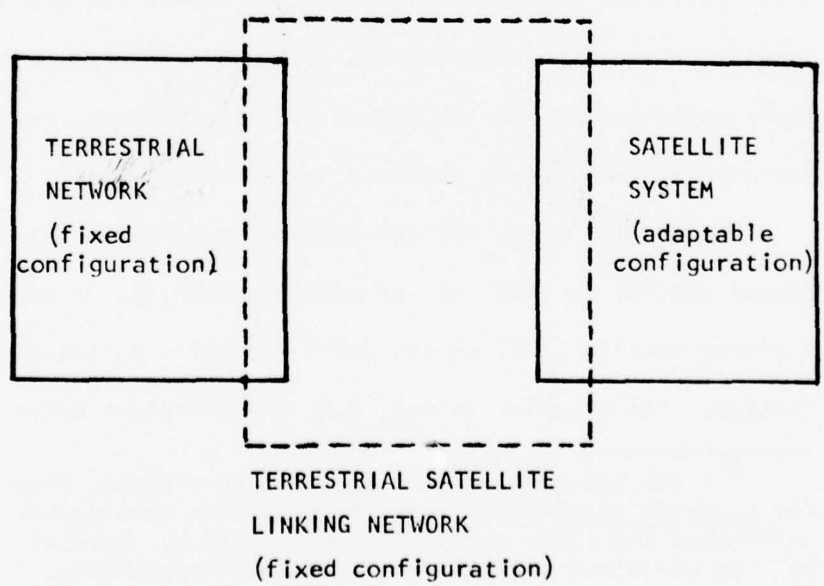


Figure 2.2. A Hybrid Communication System

In modeling the network under study, the following links may be considered:[3]

1. Pre-assigned Trunks: Such trunks may connect a transit center (or switching center) to another transit center or a ground station. These links operate between well-defined fixed points, and their capacities are fixed.

2. Fully Variable Trunks: Both the terminal nodes of such trunks are the nodes representing ground stations. Their capacities as well as their terminal nodes are variable. The total number of fully variable trunks in the network are fixed.

With the two types of trunks and two types of nodes (TC and GS), a hybrid communication network can be represented using four different symbols. An example of such a network is shown in Fig. 2.3. The circular nodes represent the transit centers (or switching centers) and the triangular nodes represent the ground stations. The solid lines represent the pre-assigned trunks; while the broken (dashed) lines represent the fully variable trunks. A location having both the transit center and the ground station in close proximity is represented by a circle inserted inside the triangle.

The routing table for the hybrid network is constructed in a manner similar to that for terrestrial networks. A country without a ground station facility can spill its calls to the closest earth station. For example, in Fig. 2.3, the switching center represented

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[3] If the economic aspects are also considered, then the need for variable destination links must also be considered. Such links arise when the links are leased by a country. However, our study will be concerned with the two types mentioned above.

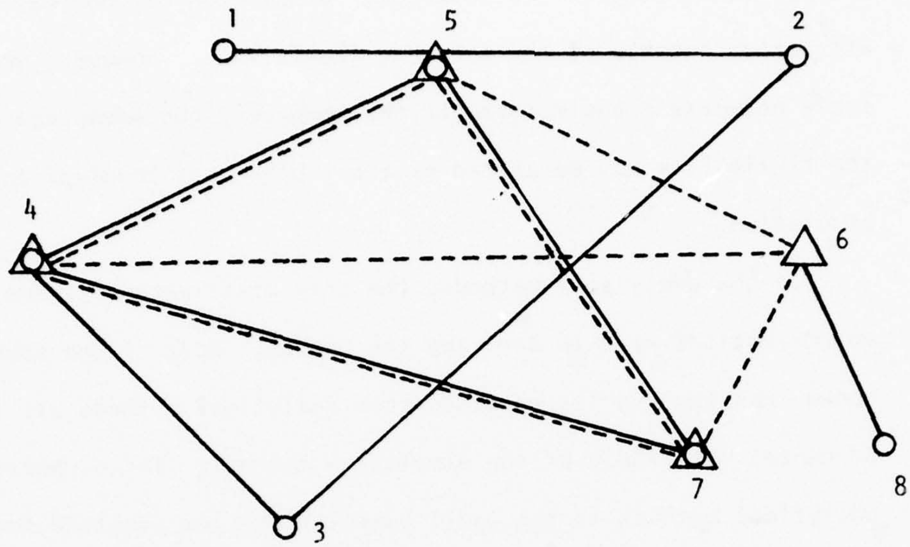

Figure 2.3. A Hybrid Communication Network

by node number 1 can spill all of its calls to the ground station numbered 5. By spilling the call, node 1 can also share the "benefits" of alternate routing.

2.4 Analysis Method

Most of the optimizing criteria for communication networks are related to improvement in the network performance, reduction in cost, etc. In this study, the optimization criteria is the improvement of the network performance. An in depth discussion of the problem is presented in the next section.

In order to improve network performance, a method of some-how estimating the performance is needed. Two kinds of methods are generally used for such studies: simulation and analytic [5]. In the former method, artificial traffic is generated on the computer, and an estimate of the system behavior is obtained. Simulation methods may look attractive because of the inherent flexibility. However, when large scale networks are simulated on the computer, the advantage due to the flexibility may be abated by the high cost involved in the simulation.

In the analytical methods, the only disadvantage is due to the approximations made in deriving the method. Most of the studies have shown that the results obtained from analytical methods are in good agreement with those of the simulation methods. The advantage of analytical methods is the small computation time required for estimating the network performance.

In the demand assignment mode of operation, it is important that a fast estimate of the network performance be obtained, so that the fully variable trunks can be re-allocated to meet the changing traffic demands. Consequently, for the problem to be presented in the next section, analytical techniques are preferred.

The analytical method used for obtaining the network performance has been adapted from [3]. An overview of this so-called single-moment method will be presented here.

In deriving the single-moment method, the following assumptions are made:

1. Call Arrival is a Poisson process
2. Call holding time has a negative exponential distribution
3. Link blocking probabilites are statistically independent
4. Nodes are non-blocking (has been stated earlier)
5. Blocked calls are cleared and do not return
6. The network is in statistical equilibrium

7. Call set-up time is negligible

8. Call arrival on any link is a Poisson process

By making the assumption that the overflow traffic is also Poisson distributed (assumption 8), we obviate the need to use the dual-moment method [5]. By virture of this assumption, the mean and the variance are equal, and the process is characterized only by the mean--hence, the name "single-moment method."

The following steps are used to determine all the node-to-node grade of services by the single moment method:

Step 1: From the given network configuration, the call control rule (OOC with spill, in our case), and the routing table, find the augmented route tree for every node pair.

Step 2: From the traffic matrix and the augmented route trees, find the link blocking probabilities using the Erlang's loss formula [6]:

$$y_i = \frac{a_i^{N_i} / N_i!}{\sum_{k=0}^{N_i} (a_i^k / k!)} \quad , \quad i = 1, 2, 3, \ldots \ell \qquad (2.1)$$

where a_i and N_i are the offered loads (in Erlangs) and the number of trunks in the ith link, respectively, and ℓ is the total number of links in the networks.

Step 3: From the link blocking probabilities found in Step 2, find the probability of each path in the augmented route tree being used to complete a call.

Step 4: Using the results of Step 3, determine the NNGOS for each node pair by[4]

$$NNGOS = 1 - \sum_j Pr\{P_j \text{ used}\} \tag{2.2}$$

From the above steps all information pertaining to the network performance can be obtained. This information may include: node grade of service, network grade of service, trunk group offered loads, load carried by each path between every node pair, etc. For a detailed description of this algorithm, refer to [3].

[4]This step will be discussed in more detail in section 4.

3. OPTIMIZATION PROBLEM

The design of circuit-switched networks involves the following basic steps:

(a) Determination of network configuration (or topology) to provide communication services to the users.

(b) Determination of trunk group (link) sizes and routing table to provide satisfactory network performance.

In the design of terrestrial communication networks, previous approaches have been based on the assumption that the network configuration is fixed. Once the network topology and trunk group sizes have been decided, there is little freedom to improve the network performance by altering the routing table. Adaptive routing techniques are available to improve the network performance [3]. But when the traffic load becomes too high, these techniques are inadequate to meet the demands. The optimization criterion used in terrestrial networks can be reduction in cost, improvement of the network performance, or the improvement in the point-to-point performance.

A satellite communication system can be used in a changing network configuration mode. In fact, to take advantage of the inherent flexibility of such systems, network adaptation should be an essential control feature. Along with the network adaptability feature is the freedom to vary the size of the variable trunks. The only restriction is that the total sum of these variable trunks must not exceed the satellite capacity.

In the hybrid communication network, therefore, there are several options available to the designer to meet a specified network performance. Adaptive routing techniques, network adaptability and the allowed changes in trunk group sizes can be used to meet an optimization criterion.

The optimization criterion that we have chosen in this report is to minimize node-to-node grade of service. The objective is to design a network for a given traffic matrix such that all the node-to-node grade of services are below a prescribed value (abbreviated BPMAX--maximum allowed blocking probability).

To give a formal statement of the problem, we make the following definitions:

G = number of terrestrial communication nodes

S = number of satellite ground stations

$\underset{\sim}{T}_\tau = [t_{ij}]_\tau$ = traffic demand matrix, where the element t_{ij} specifies the demand from node i to node j at time τ.

$\underset{\sim}{C}_\tau = [c_i]_\tau$ = link capacity vector, where the C_i element denotes the number of trunks in the ith link at time τ.

BPMAX = maximum allowable blocking probability between each node pair.

$\underset{\sim}{R}_\tau$ = routing table at time τ.

$\underset{\sim}{B} = [b_{ij}]$ = node-to-node grade of service matrix, where b_{ij} element denotes the NNGOS of i-to-j node pair.

We further assume that some of the elements of $\underset{\sim}{C}_\tau$ are fixed. These correspond to the links between a switching center and ground station or between two switching centers.

Our objective is to find a routing table and a link capacity vector such that all the node-to-node grade of services are below a prescribed BPMAX. Furthermore, in satisfying this criterion, the number of fully variable trunks used should be minimized. Mathematically,

$$\begin{aligned} C_\tau &= \{[c_j] \mid b_{ik} \le \text{BPMAX}, \min [\Sigma c_j]\} \\ R_\tau &= \{[r_{ik}] \mid b_{ik} \le \text{BPMAX}\} \end{aligned} \qquad \begin{aligned} j &= 1, 2, 3, \ldots, \ell \\ i &= 1, 2, 3, \ldots, G+S \\ k &= 1, 2, 3, \ldots, G+S \end{aligned} \tag{3.1}$$

Other optimization criteria do exist [7] but we will be concerned with the one mentioned above.

ground stations. The up-link and down-link transmission delay prohibits

To see the complexity of this problem, let us quantify some of the available options: For the S node satellite system there can be a total of $\frac{S(S-1)}{2}$ links. The number of possible link combinations is identical to the number of possible network configurations. This upper bound can be easily derived as:

$$\sum_{i=1}^{N} \binom{N}{i} = 2^{N}-1 \qquad \text{where} \qquad N \equiv \frac{S(S-1)}{2} \tag{3.2}$$

Each of the network configurations can be further modified by the large number of possible trunk assignments. Obviously, there is an astronomical number of the combinations of conceivable network topologies and trunk group sizing.

The traffic matrix and the prescribed BPMAX does reduce the number of the possible combinations. Other constraints such as satellite/ground station visibility, power sharing, uplink and downlink losses, etc. further reduce the upper bound on all the possible combinations. Despite all of these reductions the number of available options is still astronomical.

Sometimes, even if a solution to the aforementioned problem is found, there are other problems that may arise. For example, it is possible for a calculated route to use the same satellite more than once. Such a route is, of course, undesirable because of the double transmission time delay involved. Should such a problem arise, the need to re-allocate link capacities and obtain a new routing table is evident.

Finally, the algorithm for solving this network design problem must be fast. If the demand requests are monitored every $\Delta\tau$ seconds, the algorithm should be able to make the routing table and trunk assignments in less than $\Delta\tau$ seconds so that it can track the demand assignment monitor. Such an algorithm could then be used in real-time computing.

4. ALGORITHM FOR TRUNK ASSIGNMENT AND ROUTING TABLE GENERATION

4.1 Introduction

In this section we will present a heuristic algorithm for obtaining the branch capacity vector and the routing table, for a given traffic demand, that will meet the prescribed BPMAX specification. The algorithm also attempts to minimize the number of fully variable trunks used in the network.

The next two sub-sections are devoted to showing the effect of network topology and alternate routing on trunk group sizing. The ideas gathered in these sub-sections will aid in the development of the algorithm to be presented in section 4.4.

4.2 Effect of Network Topology on the Link Capacity Vector

In terrestrial communication networks it is economically unfeasible to connect a trunk group between every pair of switching centers in the network. Some of the calls have to be either spilled to another office, or to be routed through a tandem office.

In the satellite system, one of the (2^N-1) possible network configurations* is a complete graph; that is, there is a link between every pair of ground stations. Consequently, a portion of the hybrid communication network that we are concenred with can be represented by a complete graph.

An obvious question, which must be answered before any decision on the trunk group sizing algorithm is made, is the following: Is it advantageous to operate a network in a complete graph--as opposed to an incomplete graph--configuration when such a choice exists?

*See Eqn. (3.2).

A pure graph-theoretic point of view reveals that, for a n node network, the complete graph will be strongly connected as compared to an incomplete graph. Accordingly, it will be less vulnerable or more damage resistant [8].

At this time, it is not possible to give a mathematical proof, based on the single-moment method,[1] to show that the total number of trunks required to achieve a prescribed BPMAX is, in general, smaller for complete graphs than incomplete graphs. All of the network examples analysed using STARTUP [3] have validated this conjecture. Consider, for example, the five-node network of Fig. 4.1(a). Using the symbols introduced in section 2.3, the pre-assigned links are represented by solid lines, and the fully variable links by broken lines. The satellite ground stations and switching centers of the terrestrial network are grouped together. Notice that there are no terrestrial links between node pairs (1,3) and (2,5).

The BPMAX specification is 0.025--that is, all the node-to-node grade of services must be less than 2.5%. The traffic matrix for this network is given in Table 4.1. Assuming that the combined capacity of the terrestrial network is not enough to handle the traffic demand, we must assign satellite trunks (circuits) to fulfill the prescribed BPMAX requirement.

Two of the several network configurations possible are depicted in Fig. 4.1(a) and (b). In Fig. 4.1(a) we have assigned the fully variable trunks such that a complete graph is formed. In Fig. 4.1(b) we assign trunks to increase the capacity of the already existing terrestrial trunk groups.

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[1]This is a nonlinear approach to the performance analysis; based on the flow analysis approach, one could show this conjecture using the min cut-max flow argument.

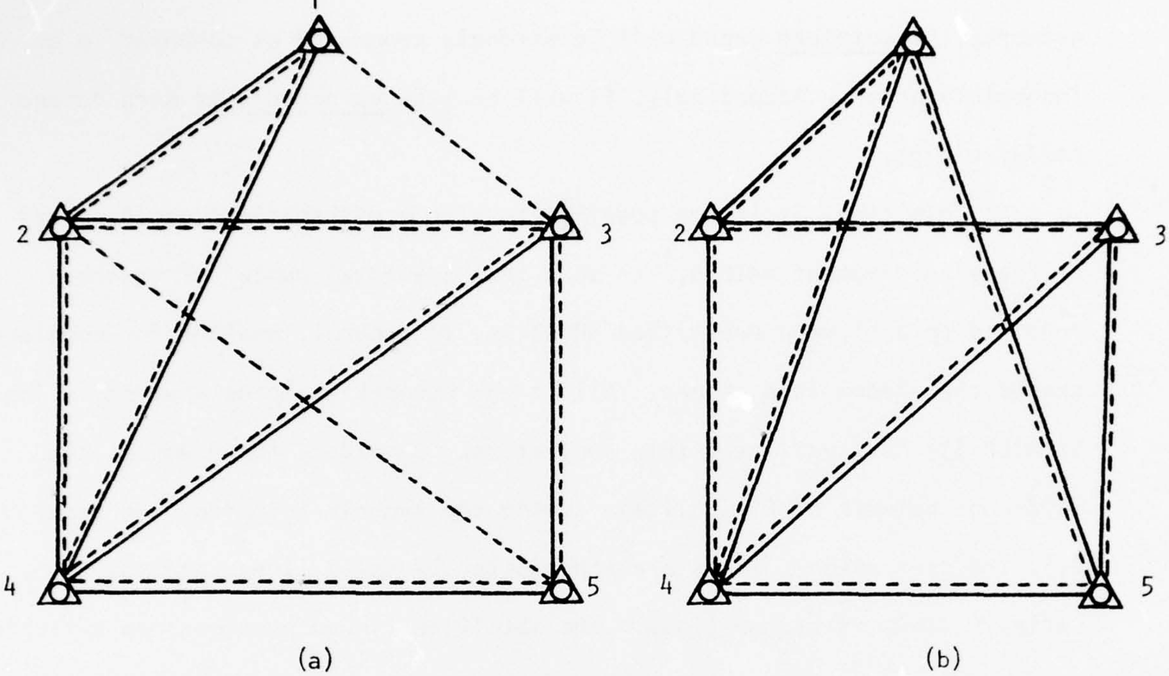

Figure 4.1. A five-node Hybrid Communication Network

Table 4.1. Traffic in Erlangs for the 5-node network of Figure 4.1

FROM \ TO	1	2	3	4	5
1		8.	12.	19.	16.
2	2.		13.	23.	12.
3	26.	19.		7.	25.
4	20.	16.	15.		19.
5	15.	12.	18.	13.	

BPMAX = 0.025

Clearly, there are several different combined[2] branch capacity vectors that can be chosen to meet the BPMAX specification. One of such vectors is given in Table 4.2(b). Based on the traffic matrix and the trunk group sizes, we obtain the routing table depicted in Table 4.2(a) such that all the NNGOS are less than 0.025. The network performance using STARTUP is shown in Fig. 4.2.

Table 4.2 Routing Table and Trunk Group Information for Fig. 4.1(a)

FROM \ TO	1	2	3	4	5
1		2,5	3,2	4,2	5,2
2	1,5		3,1	4,1	5,1
3	1,2	2,1		4,5	5,4
4	1,2	2,1	3,5		5,3
5	1,2	2,1	3,2	4,2	

(a) Routing Table

Link Number	Terminal Nodes	Number of Trunks
1	1-2	31
2	1-4	44
3	1-5	36
4	1-3	41
5	2-3	39
6	2-4	45
7	2-5	33
8	3-4	28
9	3-5	48
10	4-5	38

(b) Trunk Group Information

[2] By <u>combined</u> we mean the sum of the pre-assigned and fully variable trunks present in a link.

ACTUAL TRUNK GROUP PERFORMANCE (BASED ON ORIGINAL ROUTING TABLE)

LINK NUMBER	OFFERED LOAD(CCS)	TRUNK GROUP ACTUAL GOS	TRUNK GROUP RELIABILITY
1	974.9	.067885	.932115
2	1522.6	.090474	.909526
3	1198.2	.083114	.916886
4	1472.2	.113341	.886659
5	1349.9	.097073	.902927
6	1561.3	.090614	.909386
7	1063.1	.074000	.926000
8	920.8	.092755	.907245
9	1670.5	.088969	.911031
10	1291.6	.089710	.910290

NETWORK PERFORMANCE (BASED ON ORIGINAL ROUTING TABLE)

SOURCE	DESTINATION	BLOCKING PROBABILITY	WEIGHTED BLOCKING PROBABILITY
1	2	1.025E-02	1.025E-02
1	3	1.795E-02	1.795E-02
1	4	1.378E-02	1.378E-02
1	5	1.138E-02	1.138E-02
2	1	1.025E-02	1.025E-02
2	3	1.685E-02	1.685E-02
2	4	1.379E-02	1.379E-02
2	5	1.076E-02	1.076E-02
3	1	1.795E-02	1.795E-02
3	2	1.685E-02	1.685E-02
3	4	1.583E-02	1.583E-02
3	5	1.549E-02	1.549E-02
4	1	1.378E-02	1.378E-02
4	2	1.379E-02	1.379E-02
4	3	1.583E-02	1.583E-02
4	5	1.556E-02	1.556E-02
5	1	1.138E-02	1.138E-02
5	2	1.076E-02	1.076E-02
5	3	1.458E-02	1.458E-02
5	4	1.417E-02	1.417E-02

NODE GRADE OF SERVICE (BASED ON ORIGNAL ROUTING TABLE)

NODE NUMBER	ORIGINATING LOAD (CCS)	NODE GRADE OF SERVICE	NODE RELIABILITY
1	1980.0	.013478	.986522
2	1800.0	.013716	.986284
3	2772.0	.016687	.983313
4	2520.0	.014708	.985292
5	2088.0	.012868	.987132

LARGEST WEIGHTED NNGOS(= 1.795E-02) IS FROM 3 TO 1

MAXIMUM ALLOWABLE BLOCKING PROBABILITY(BPMAX) .025

NETWORK GRADE OF SERVICE IS .01448
WEIGHTED NETWORK GRADE OF SERVICE IS .01448

Figure 4.2. Performance for the Network of Fig. 4.1(a).

For the network configuration of Fig. 4.1(b), there are again many possible branch capacity vectors and routing tables that can be used. One of such combinations is given in Tables 4.3(a) and (b). The results of the analysis of this configuration are given in Fig. 4.3.

Table 4.3 Routing Table and Trunk Group Information for the Network of Fig. 4.1(b)

FROM \ TO	1	2	3	4	5
1		4,3,5	3,5	4,5	5,3
2	5,4,3		3,4,5	4,5	5,3
3	1,5	2,5		5,1,2	5,1
4	1,5	2,5	5,2,1		5,2
5	1,3	2,4	3,1	4,1	

(a) Routing Table

Link Number	Terminal Nodes	Number of Trunks
1	1-4	48
2	1-5	45
3	1-3	49
4	2-3	39
5	2-4	53
6	2-5	34
7	3-5	72
8	4-5	63

(b) Trunk Group Information

ACTUAL TRUNK GROUP PERFORMANCE (BASED ON ORIGINAL ROUTING TABLE)

LINK NUMBER	OFFERED LOAD(CCS)	TRUNK GROUP ACTUAL GOS	TRUNK GROUP RELIABILITY
1	1765.9	.118703	.881297
2	1635.9	.115453	.884547
3	1732.2	.095715	.904285
4	1341.4	.093845	.906155
5	1891.3	.096962	.903038
6	1176.3	.105053	.894947
7	2650.4	.101344	.898656
8	2209.6	.080406	.919594

NETWORK PERFORMANCE (BASED ON ORIGINAL ROUTING TABLE)

SOURCE	DESTINATION	BLOCKING PROBABILITY	WEIGHTED BLOCKING PROBABILITY
1	2	7.682E-03	7.682E-03
1	3	1.963E-02	1.963E-02
1	4	2.215E-02	2.215E-02
1	5	2.163E-02	2.163E-02
2	1	7.682E-03	7.682E-03
2	3	1.101E-02	1.101E-02
2	4	1.716E-02	1.716E-02
2	5	1.951E-02	1.951E-02
3	1	1.963E-02	1.963E-02
3	2	1.837E-02	1.837E-02
3	4	6.405E-03	6.405E-03
3	5	2.028E-02	2.028E-02
4	1	2.215E-02	2.215E-02
4	2	1.716E-02	1.716E-02
4	3	6.405E-03	6.405E-03
4	5	1.542E-02	1.542E-02
5	1	2.163E-02	2.163E-02
5	2	1.781E-02	1.781E-02
5	3	2.028E-02	2.028E-02
5	4	1.773E-02	1.773E-02

NODE GRADE OF SERVICE (BASED ON URIGNAL ROUTING TABLE)

NODE NUMBER	ORIGINATING LOAD (CCS)	NODE GRADE OF SERVICE	NODE RELIABILITY
1	1980.0	.019344	.980656
2	1800.0	.015747	.984253
3	2772.0	.018328	.981672
4	2520.0	.015810	.984190
5	2088.0	.019547	.980453

LARGEST WEIGHTED NNGOS(= 2.215E-02) IS FROM 4 TO 1

MAXIMUM ALLOWABLE BLOCKING PROBABILITY(BPMAX) .025

NETWORK GRADE OF SERVICE IS .01775
WEIGHTED NETWORK GRADE OF SERVICE IS .01775

Figure 4.3. Performance for the Network of Fig. 4.1(b).

From Figs. 4.2 and 4.3, we note that the BPMAX criterion is met in both the cases. The total number of trunks needed for the network configurations of Figs. 4.1(a) and (b) is 383 and 403, respectively. Thus, there is a saving of trunks if we use the complete graph configuration; furthermore, the network grade of service is smaller for this case.

With this small network example, we have illustrated the advantage accrued in choosing a complete graph topology over another topology. As mentioned earlier--even for this simple network--there are several choices for the branch capacity vectors for a given configuration. In all of the examples investigated, our results indicate that the complete graph configuration always uses fewer trunks than another configuration to meet the same BPMAX specification. A derivation of this conjecture should be a part of any future research.

4.3 Effect of Alternate Routing on the Link Capacity Vector

In this section, we will explore the effect of alternate routing on trunk group sizing. If we were concenred only with the satellite system, such an investigation would be unwarranted; for, the inherent delay in the uplink and downlink transmission makes the use of alternate routing strategy undesirable. However, in the hybrid system alternate routing is feasible. We can assume that a call is routed in a manner such that is uses the satellite "trunk" at most once in its propagation from the source to the destination. Consequently, it is meaningful to investigate the behavior of NNGOS for different trunk group sizes and routing strategies.

The relationship between the offered traffic and node-to-node grade of services for fixed trunk group sizes and different routing schemes has been reported in [4]. Grandjean's results show that the "nondirect" routing with more alternate paths is more sensitive to overload than alternate routing with fewer paths. Direct routing has the least sensitivity with regard to overload.

In our problem, we are not only concerned with selecting a routing strategy, but also minimizing the number of trunks required to meet a prescribed BPMAX value. An obvious question that arises is: For a prescribed traffic matrix, what is the relationship between the node-to-node grade of services and varying link sizes for different routing schemes?

We will show this relationship using an idealized network example. Consider the five-node network of Fig. 4.4. The network is completely symmetrical with regard to traffic and routing strategies; that is, the traffic between each node pair is equal and every link has the same offered and carried loads. The number of trunks assigned to each link is the same.

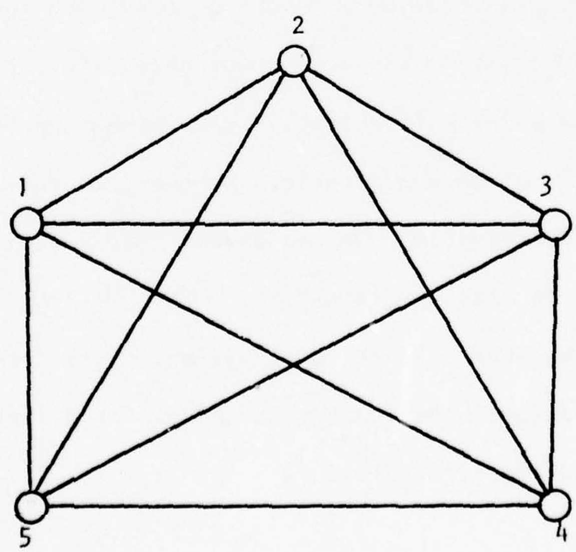

Figure 4.4. A five-node symmetrical network

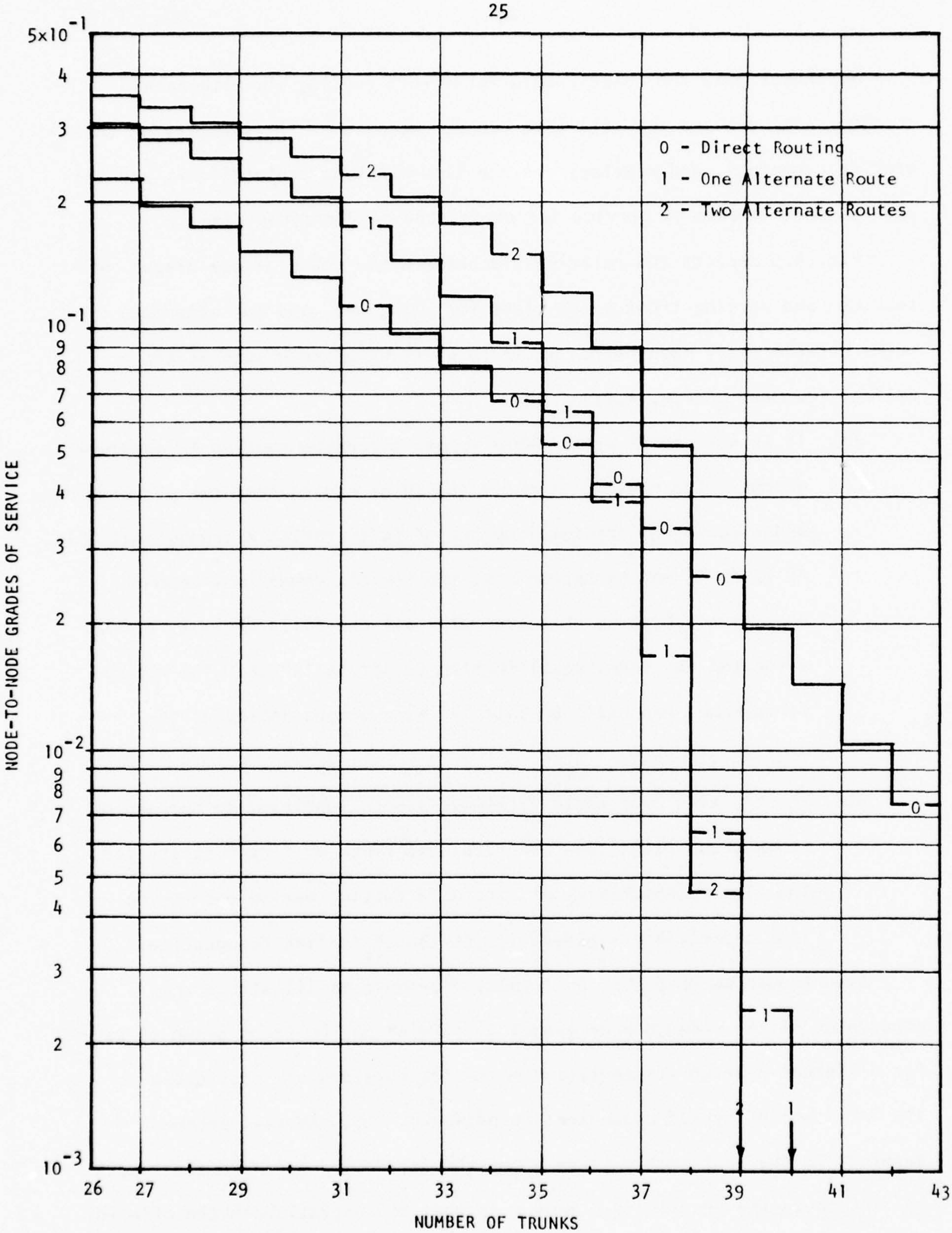

Fig. 4.5 NNGOS vs. Number of Trunks for the network of Fig. 4.4 (Traffic = 15 Erlangs)

We investigate the relationship for direct routing and alternate routing with one and two alternate paths. Because of the symmetry in the traffic, topology, and routing, all the link blocking probabilities and the node-to-node grades of service for every node pair are the same.

Fig. 4.5 depicts the relationship between the node-to-node grades of service and varying trunk group sizes for zero, one, and two alternate paths between every node pair. It is interesting to note some of the salient features of Fig. 4.5:

(a) It is not always preferrable to use alternate routing in complete graphs. The routing strategy should be chosen from the prescribed BPMAX value and the total number of fully variable trunks available. At times it may so happen that the traffic demand may become excessively high so that even with all the fully variable trunks allocated the BPMAX specification is not satisfied. The network performance can still be improved by a proper choice of the routing strategy.

(b) The two alternate route strategy is more sensitive to decreasing trunk group sizes than one alternate route or direct strategies. The higher sensitivity of alternate routing due to decreasing link capacities is generally attributed to link "congestion."

Even though we have used an idealized example to illustrate the dependence of the node-to-node grades of service on the trunk group sizes for different routing strategies, the results obtained are applicable to the "real world" hybrid communication networks, too. As will become apparent in the next section, our algorithm for demand assignment attempts to distribute the traffic in a manner so that the variation in the offered loads to the links is minimal.

4.3 An Algorithm

A search of the literature reveals no existing work dealing with the solution to the problem formulated in section 3--that of determining the trunk group sizes and routing table generation simultaneously. Previous works [3,9] have treated these as disjoint problems. Reference [3] gives an algorithm for obtaining a routing table that satisfies a prescribed BPMAX specification, assuming that the network configuration and trunk group sizes are fixed. On the other hand, [9] uses an iterative scheme to obtain the branch capacity vector for a pre-specified routing table.

In this section, we will present an algorithm that will generate a branch capacity vector and routing table for a network with adaptable configuration, such that all the NNGOS are below a prescribed BPMAX value. Of course, if the traffic demands get excessively high in relation to the network call-carrying capacity, this algorithm fails to meet the BPMAX criterion.

The research reported herein is a preliminary investigation of the routing problem in hybrid communication systems. Consequently, no rigorous derivations of the concepts used are available at this point. We shall, however, present heuristic arguments to justify the steps involved.

Our aim is to determine an algorithmic way of designing networks with "semi-adaptable" configurations such that all the NNGOS are below an upper bound. In order to facilitate the understanding of the various steps, and their order, in the algorithm, we will show how the NNGOS are calculated and their dependence on the link grade of services, traffic matrix, and the routing table.

4.3.1 Factors that Influence NNGOS Values

The complexity inherent in the hybrid communication network routing problem was presented in section 3. This section reveals some additional difficulties involved in the solution to the problem we have formulated. A systematic approach can be devised after the basic complexities in the problem are understood.

The network performance (determination of the NNGOS) based on the single-moment method can be obtained provided we have the following information:

1. Network Topology

   (a) link-node incidence description (b) link sizes

2. Routing Table
3. Call Control Rule
4. Traffic Matrix

In our case, the desired network performance, (3) and (4) are known; the problem is to find (1) and (2). Clearly, we note that we have an extra unknown in our problem. Thus, an obvious question is: knowing the desired network performance (specified by the BPMAX requirement), (3) and (4) above, is it possible to find (1) and (2)? This question can be answered after we develop the method for determining the NNGOS for all node pairs:

We first define the following notation:

$\underline{y} \triangleq$ vector of link blocking probabilities

$\underline{x} \triangleq \underline{1} - \underline{y} \triangleq$ vector of link reliabilities

$\underline{a}' \triangleq$ vector of link carried loads

$\underline{a} \triangleq$ vector of link offered loads

$\underline{c} \triangleq$ branch capacity vector

In order to determine the NNGOS for all node pairs, the $\underline{y}$ vector, the call control rule, and the routing table must be known. The $\underline{y}$ vector is found by using the Erlang loss formula [6], where each element in the vector is given by:

$$y_i = \frac{\dfrac{a_i^{c_i}}{c_i!}}{\sum_{k=0}^{c_i} (a_i^k/k!)} \quad , \qquad i = 1, 2, 3, \ldots, \ell \tag{4.1}$$

where a_i and c_i are the offered load and link capacity of the ith link, respectively. ℓ is the number of links in the network.

Clearly, the link grades of service are not invariant. The value y_i is a nonlinear function of the offered loads, a_i, and the number of trunks, c_i, in each of the links in the network. In vector notation, (4.1) can be expressed as:

$$\underline{y} = \underline{F}_1(\underline{a}, \underline{c}) \tag{4.2}$$

The trunk group offered load vector, in turn, depends on the $\underline{y}$ vector and the <u>traffic distribution vector</u> $\underline{t}$. That is,

$$\underline{a} = \underline{F}_2(\underline{y}, \underline{t}) \tag{4.3}$$

The traffic distribution vector is dependent upon the traffic matrix, routing table, call control rule, and link blocking probabilities. Both $\underline{F}_1$ and $\underline{F}_2$ are non-linear functions.

Knowing the $\underline{y}$ vector, the NNGOS can be found from the augmented route trees[1] for each node pair. For example, consider the augmented route tree shown in Fig. 4.6.

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[1] These are defined by the routing table.

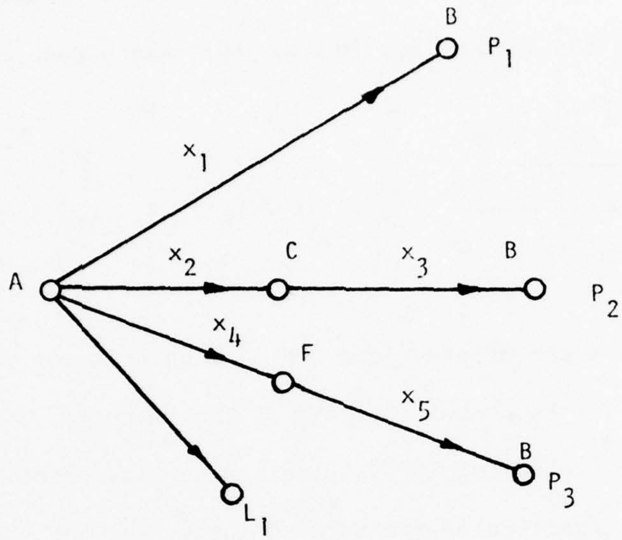

Figure 4.6 An Augmented Route Tree

The NNGOS is found as follows:

$$\Pr\{P_1 \text{ used}\} = x_1$$

$$\Pr\{P_2 \text{ used}\} = x_2x_3(1-x_1)$$

$$\Pr\{P_3 \text{ used}\} = x_4x_5(1-x_1)(1-x_2x_3)$$

Then,

$$\begin{aligned} \text{NNGOS}_{A\to B} &= 1 - \sum_j \Pr\{P_j \text{ used}\} \\ &= 1 - [x_1 + x_2x_3(1-x_1) + x_4x_5(1-x_1)(1-x_2x_3)] \end{aligned} \tag{4.4}$$

Equation (4.4) is again a nonlinear algebraic equation. From the nonlinear relationships between $\underline{y}$, $\underline{a}$, $\underline{t}$, and node-to-node grade of services, it should be apparent that a two-step algorithm--one step to arbitrarily assign trunks and the second to generate a routing table--does not provide an answer to the problem posed in section 3. The need *for simultaneously* generating a routing table and assigning trunks is evident.

We are now in a position to answer the question posed at the beginning of this sub-section: from the coupling between equations (4.2) and (4.3), it is evident that information regarding the desired network performance, call control rule and the traffic matrix are not sufficient for the determination of the branch capacity vector and the routing table independently. To determine the branch capacity vector, the vector of offered loads to the links must be known. The need to make some approximations or assumptions is evident.

The offered load to various links is controlled by the routing table and the Erlang loss formula. Prior to making any assumptions, we note some of the properties of the Erlang B formula [6, 10]:

The elements of the carried load vector, $\underline{a}'$, can be obtained as follows:

$$a'_i = a(1-y_i) \tag{4.5}$$

The utilization factor ρ_i of the ith trunk group in steady state is defined as:

$$\rho_i = \frac{a'_i}{S_i} \tag{4.6}$$

Investigations [6] into equations (4.1), (4.5), and (4.6) have led to the conclusion that larger trunk groups are more efficient than smaller ones; that is, as the number of trunks in a link is increased and the offered load is increased such that the probability of blocking remains constant, the utilization factor increases. The disadvantage of a link having a larger utilization factor is its higher sensitivity to overload. We shall use some of the above ideas in devising a scheme for distributing the loads to various trunk groups. Rather than making some trunk groups extremely large and some extremely small, we shall attempt to distribute the load such that the variation in the offered loads and link capacities is minimal. The utilization factor of all the links will then be nearly equal. By considering the flow-problem, rather than the performance analysis problem, Paz [11] has shown that improved traffic handling capability is obtained if the variation in normalized branch flows (ratio of traffic flow and capacity of a link) is minimal.

4.3.2 Description of the Algorithm

The algoirthm described in this section is basically a one-pass scheme for obtaining the branch capacity vector and routing table, given a BPMAX value and the traffic demand matrix. Even though a nonlinear analysis method is used for allocating the fully variable trunks, we shall nevertheless make use of some linear approximations. Provisions have been made to detect and correct the discrepencies between the actual and the desired network performance that may arise due to these approximations.

In obtaining this algorithm, the following assumption [see section 2.4 for others] is made:

- The time required to search, size, and later release a trunk is negligible.

The algorithm can be programmed on a digital computer. An algorithmic flow-chart is shown in Fig. 4.7. As indicated in Fig. 4.7, there are basically six steps involved.

To facilitate the understanding of the various steps involved, we will use the seven-node hybrid network of Fig. 4.8 to illustrate some of the intermediate results. Again, the terrestrial links are represented by solid lines. The satellite ground stations (GS) are represented by

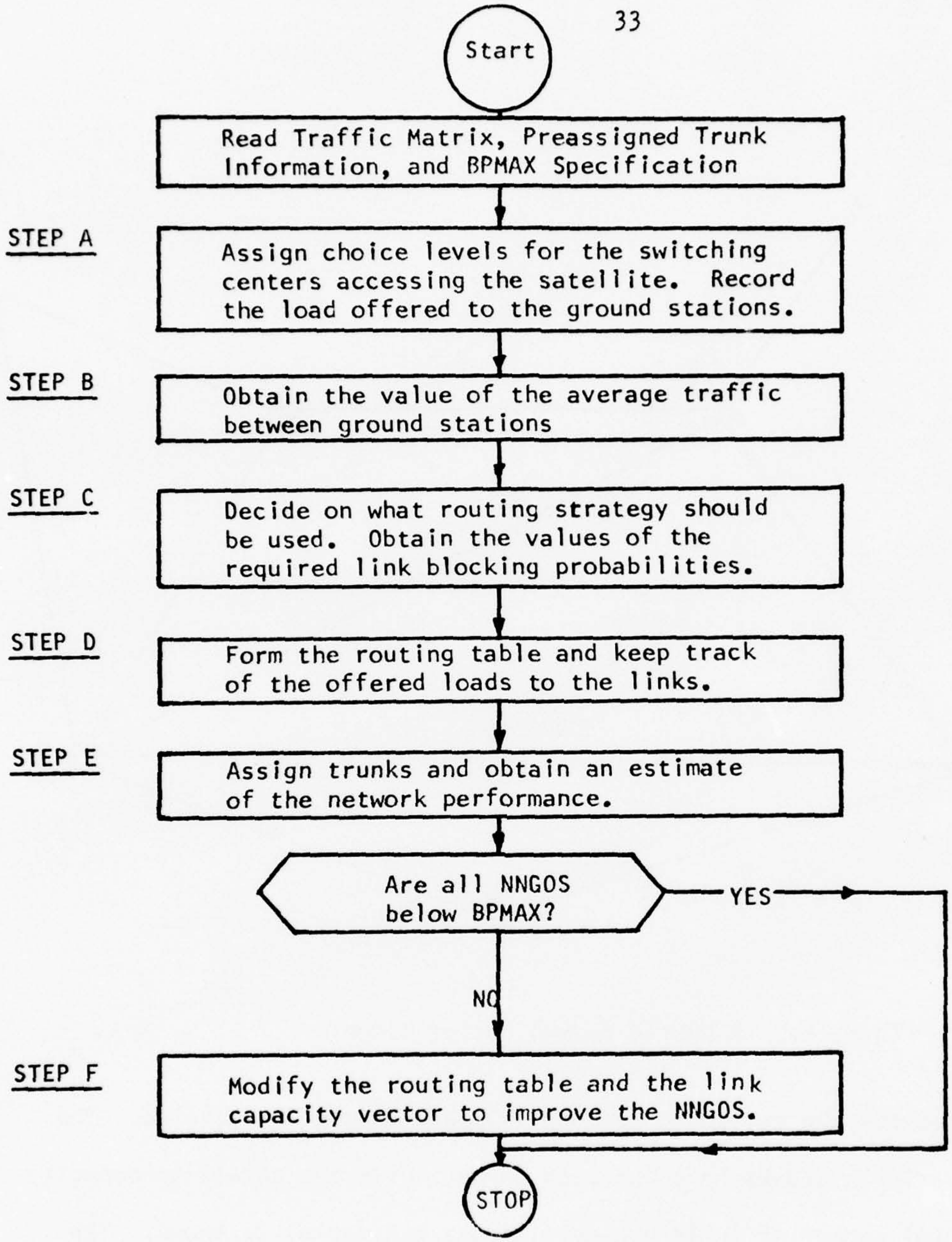

Figure 4.7. Algorithm for generating a Routing Table and a Link Capacity vector for a given BPMAX and Traffic demands.

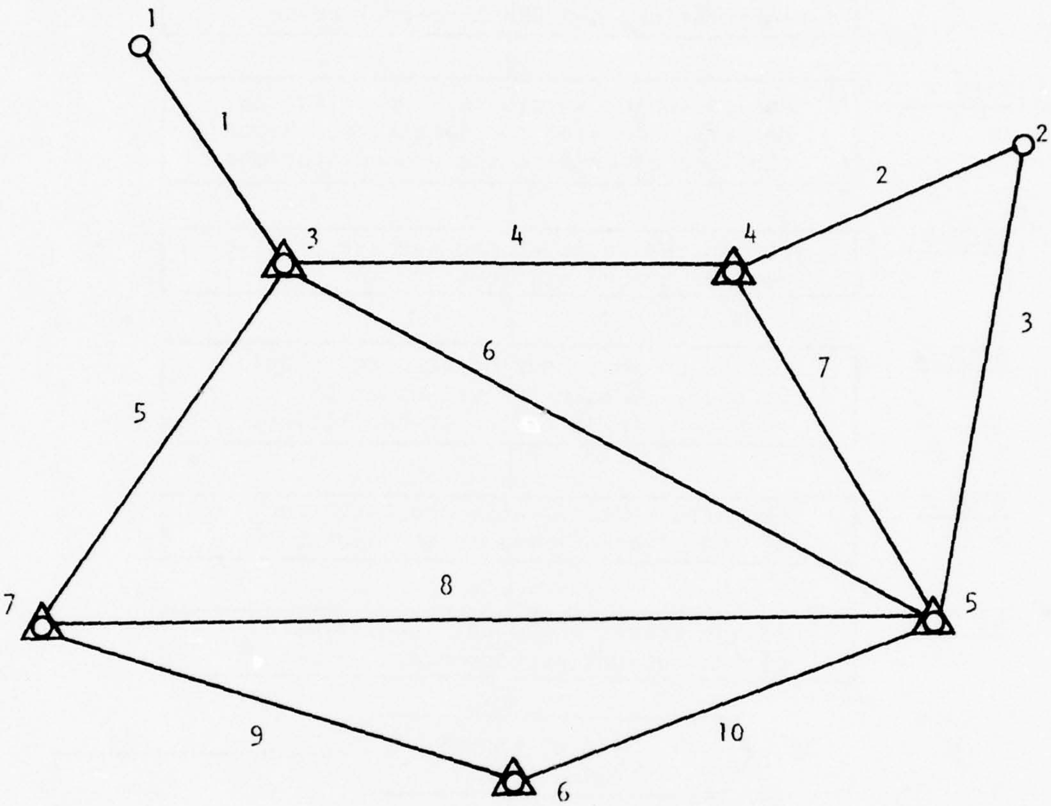

Figure 4.8 A Hybrid Communication Network

triangles and the switching centers (TC) are denoted by circles. The fully variable trunks have not been shown. Only the satellite capacity (the total number of fully variable trunks available) is known. The traffic matrix and pre-assigned trunk informations are given in Table 4.4(a) and (b), respectively.

We will now make some definitions and assumptions:

• <u>Def</u>: <u>Satellite Section</u>: The part of the hybrid system comprising of the ground stations is defined as the satellite section.

Table 4.4

FROM \ TO	1	2	3	4	5	6	7
1		4.2	1.4	1.4	2.8	1.4	1.4
2	1.4		1.4	2.1	3.5	1.4	2.8
3	1.4	2.1		5.6	4.2	7.0	2.8
4	2.1	1.4	2.8		8.4	1.4	1.4
5	2.8	1.4	4.9	2.8		6.3	2.8
6	1.4	2.8	3.5	2.8	2.8		4.2
7	0.7	1.4	1.4	2.8	7.7	6.3	

(a) Traffic Matrix in Erlangs for the Hybrid Network of Fig. 4.8.

Link Number	Terminal Nodes	Number of Trunks
1	1-3	35
2	2-4	28
3	2-5	25
4	3-4	15
5	3-7	8
6	3-5	15
7	4-5	13
8	5-7	14
9	6-7	10
10	5-6	6

Number of fully variable trunks = 80

(b) Pre-assigned Trunk Group Information for the Hybrid Network of Fig. 4.8

• Def: Terrestrial Section: The part of the hybrid system comprising of the switching centers is defined as the terrestrial section.

A switching center and a ground station in close proximity, represented by Δ, is considered part of the satellite section. Thus, in Fig. 4.8, nodes 1 and 2 comprise the satellite section and nodes 3 through 7 comprise the satellite section.

• Def: Augmented Traffic Matrix $[t_{ij}]$ defines the total traffic between node pairs in the staellite section after the decision to spill traffic to different nodes has been made.

• $tr_{\ell \to m} \equiv$ traffic originating at node ℓ and destined for node m defined by the input traffic matrix information.

The algorithm for "demand assignment" in hybrid communication networks comprises of the following steps:

STEP A: This step involves the computation of the total traffic offered to the satellite section and some of the links with fixed capacities by the terrestrial section. This computation is easily made from the information[2] provided by the desired network performance, traffic matrix, node-link incidence relationship of the fixed section of the network. For example, consider a switching center with node degree exactly one. Then all of the traffic originating at this node must be routed on this link. Similarly, the traffic destined to this node must also travel

[2]We will assume the call control rule is known. It is OOC in our case.

the same link. The general structure of the augmented route trees for such cases are shown in Fig. 4.9(a) and (b).

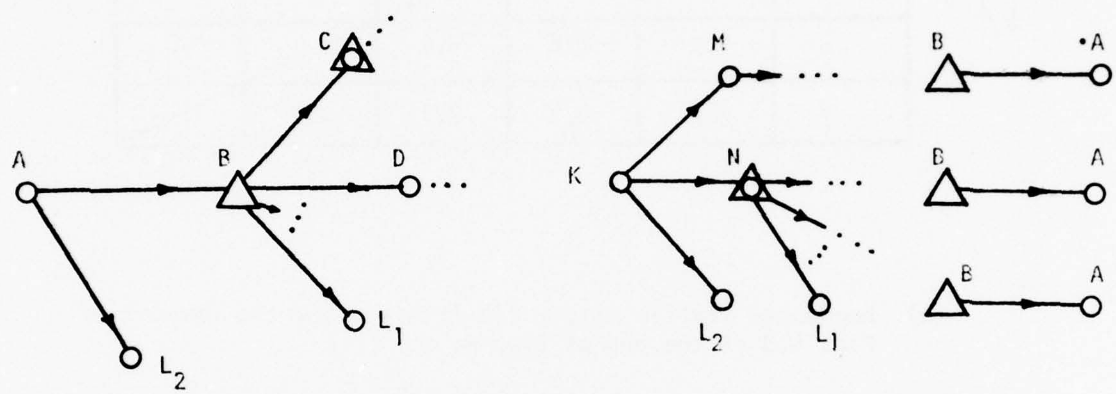

Figure 4.9 Augmented Route Trees showing (a) traffic away from node A (with node degree 1), (b) traffic to node A.

The approximate offered load* to link connecting A-B in Fig. 4.9 is given by:

$$a_{A-B} = \sum_i tr_{A\to i} + \sum_i tr_{i\to A} \quad , \ i \neq A \qquad (4.7)$$

Once a_{A-B} is known, the blocking probability of link A-B can be computed by Erlang's formula. The additional traffic that node B must route is given by:

$$\hat{t}_{B\to i} = x_{A-B}(tr_{A\to i}) \quad , \quad i \neq A \qquad (4.8)$$

where $\hat{t}_{B\to i}$ is the additional traffic originating at node B and destined from node i. Similarly, we can compute the additional traffic to the other ground stations.

*This approximation is valid for small values of BPMAX--which is quite often the case we deal with.

Table 4.5

FROM \ TO	3	4	5	6	7
3		13.3	7.0	8.4	4.2
4	9.1		8.4	1.4	4.2
5	7.7	2.8		7.7	2.8
6	4.9	2.8	5.6		4.2
7	2.1	4.2	7.7	6.3	

(a) Augmented Traffic Matrix (in Erlangs) for the network of Fig. 4.8 at the end of Step A.

FROM \ TO	1	2	3	4	5	6	7
1		3*	3	3*	3*	3*	3*
2	4		4*	4	5	5*	4*
3	1	4					
4	3	2					
5	3	2					
6	3	5					
7	3	4					

* denotes a spill switch

(b) Partial Routing Table for the network of Fig. 4.8 at the end of Step A.

In the case of SC's with node degree greater than one, the traffic offered to each of the links incident to it is in direct proportion to their capacities (linear approximation). The additional traffic that the ground stations must route can again be found by using eqn. (4.8).

When the decision to spill the traffic to the different ground stations has been made, the appropriate blocks in the routing table are filled with these choice levels.

To illustrate the use of this step, refer to the network of Fig. 4.8. The approximate+ values for the elements of the augmented traffic matrix are depicted in Table 4.5(a). The routing table at the end of this step is given in Table 4.5(b).

<u>STEP B</u>: From the augmented traffic matrix, t, for the satellite section, compute the average load between a node pair:

$$\langle t \rangle = \frac{1}{S(S-1)} \sum_i \sum_j t_{i\to j} \qquad \begin{array}{l} i \neq j \\ i = 1, 2, \ldots, S \\ j = 1, 2, \ldots, S \end{array} \qquad (4.9)$$

$$t_{i\to j} \triangleq tr_{i\to j} + \hat{t}_{i\to j} \,,$$

where S is the number of ground stations. Thus for the network of Fig. 4.8:

$$\langle t \rangle \doteq 5.74 \text{ Erlangs.}$$

<u>STEP C</u>: In this step we make an important assumption: <u>the reliabilties of all the GS-to-GS links are equal[3].</u> Based on this assumption we can compute the link reliability required to meet the prescribed BPMAX value.

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[3] At the completion of this algorithm, these may turn out to be unequal.

+These values are approximate because some of the traffic is lost at the switching center accessing the ground station. Consequently, some of these element values are larger than what the analysis will depict.

We compute the link reliabilties for different routing strategies as follows[4]:

Direct Routing: $x_0 = 1 - \mathrm{BPMAX}$

One Alternate Route: $x_1 + x_1^2(1-x_1) = 1 - \mathrm{BPMAX}$ (4.10)

Two Alternate Routes: $x_2 + x_2^2(1-x_2)(2-x_2^2) = 1 - \mathrm{BPMAX}$

Equations (4.10) are nonlinear algebriac equations, and can be easily solved by any fixed-point iteration method [12].

From the required values of x_0, x_1, and x_2, we can compute the average number of trunks per link required:

$$s_i = a_i\left[\frac{y_i(s_i-1,a_k)}{y_i(s_i,\, a_k)}\right]\{1 - y_i(s_i,\, a_k)\} \qquad k, i = 0, 1, 2 \tag{4.11}$$

where $S_i \triangleq$ number of trunks required in a link when the <u>offered</u> load is a_k and the required blocking probability is $y_i (\triangleq 1-x_i)$.

$y(s_i-1,a_k)$ can be computed from equation (4.1)

and

$a_0 \equiv 2\langle t\rangle$

$a_1 \equiv 2[1 + x_1(1-x_1)]\langle t\rangle$

$a_2 \equiv 2[1 + x_2(1-x_2)(2-x_2^2)]\langle t\rangle$

Equation (4.11) must again be solved using the fixed-point iteration approach. It should be noted that s_i is an integer; consequently a proper error criterion must be set in the iteration algorithm. The behavior of the

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[4]Equations (4.10) assume that the alternate paths are comprised of two links. Since we are dealing with a complete graph (satellite section only), this assumption is quite valid.

s_i vs. NNGOS curve will be similar to Fig. 4.5 for a given traffic matrix. The choice of a routing strategy can be easily made once we have all the s_i's and information on the total number of variable trunks.

The purpose of Step C is to give an estimate of how many fully variable trunks, in addition to the preassigned trunks, will be required to satisfy the BPMAX criterion. This step views the satellite section of the network to be symmetrical in routing, traffic matrix, and network configuration. Such an estimate is generally optimistic and, consequently, in the actual network, the number of additional trunks required are larger. But, nevertheless, this step serves its purpose by helping to decide on a routing strategy.

STEP D: Once the decision on how many alternate routes will be used has been made in Step C, the next step is to form the routing table and keep an account of how much load will be offered to the links connecting two ground stations.

To help set a criterion for forming the routing table, we will make use of the results given in section 4.2. Also, the fact that larger capacity links (with larger offered load) are more efficient than the smaller capacity links will be used.

A two-step procedure is folowed for assigning choice levels in the routing table. These steps are:

Step 1: In this step, the first choice level for routing of calls from ground station-to-ground station is selected. Since one of the configurations available is the complete graph, an obvious candidate for first choice level in the routing table is the direct choice. So the first choice for routing calls between two ground stations is the direct link. We fill up the routing table accordingly.

The offered load to the links connecting every pair of ground stations in the network is also recorded. The offered load to a link connecting two ground stations i and j is given by:

$$a^{p1}_{i\to j} = t_{i\to j} + t_{j\to i} \qquad (4.12)$$

where the superscript on a is used to denote that it is the offered load due to path 1. If a direct routing strategy was chosen in step C, step E is performed. Otherwise, we perform the following step:

Step 2: In this step we choose the second and third choices of the routing table for the GS-to-GS calls. To facilitate the assignment of these choice levels, we have established the following criterion:

The proposed alternate destination for any block in the routing table should be such that it tends to minimize the variation in the offered load to links connecting every pair of ground stations.

This criterion can be easily implemented in an algorithmic manner. The offered loads found in step 1 above can be arranged in an ascending or descending order. The second choice levels are selected such that the links with smaller offered loads from step 1 are used on these paths. The load offered to a trunk group due to the first alternate path can be easily computed. For example, if the calls from node i to node k use the link ℓ_{i-j} on its second path, then the additional offered load, $a^{p2}{}_{i\to j}$, is given by:

$$a^{p2}{}_{i\to j} = x(1-x)\ t_{i\to k} \qquad (4.13)$$

where x was found in step C.

If the same link appears on the second path of other node pairs, the additional offered loads are computed by using eqn. (4.13).

Similarly, if the calls from node n to node k use the link ℓ_{n-k} on its third choice level, then the additional offered load, $a^{p3}_{n\to k}$ is given by:

$$a^{p3}_{n\to k} = x(1-x)(2-x^2)\,t_{n\to k} \qquad (4.14)$$

Thus, in step 2, we assign second and third choice levels (depending upon which routing strategy we have selected) and keep an account of the offered loads of each of the links present between two ground stations. The combined offered load for link ℓ_{i-j} is given by:

$$a_{i-j} = a^{p1}_{i\leftrightarrow j} + \Sigma\, a^{p2}_{i\leftrightarrow j} + \Sigma\, a^{p3}_{i\leftrightarrow j} \qquad (4.15)$$

where the summation is used to emphasize the fact that the same link can appear on the second and/or third paths of different node pairs more than once; $\leftrightarrow$ is because of bi-directional link assumption.

We illustrate this step by once again referring to the network of Fig. 4.8. The routing table and the vector of <u>estimated</u> offered loads to the trunk groups at the end of this step are shown in Table 4.6.

Table 4.6

FROM \ TO	1	2	3	4	5	6	7
1		3*	3	3*	3*	3*	3*
2	4*		4*	4	5	5*	4*
3	1	4		4,7	5,7	6,7	7,4
4	3	2	3,7		5,6	6,7	7,6
5	3	2	3,7	4,6		6,4	7,4
6	3	5	3,7	4,7	5,4		7,4
7	3	4	3,4	4,3	5,4	6,3	

(a) Routing Table at the end of Step D

Link Number	Terminal Nodes	Offered Load (Erlangs) Path 1	Path 2	Total Offered Load* Erlangs	(CCS)
4	3-4	21.	8.4	21.8	(784.8)
5	3-7	6.3	44.1	10.4	(374.4)
6	3-5	14.7	0	14.7	(529.2)
7	4-5	11.2	22.8	13.3	(478.8)
8	5-7	10.5	11.9	11.6	(417.6)
9	6-7	10.5	20.3	12.4	(446.4)
10	5-6	13.3	11.2	14.3	(514.8)
11	4-7	8.4	35.5	11.7	(421.2)
12	4-6	4.2	30.1	7.0	(252.)
13	3-6	11.9	7.7	12.6	(453.6)

*Total Offered Load = (Offered Load on Path 1) +

$x \cdot (1-x) \cdot$ (Offered load on Path 2), $x = .897$

(b) Estimated Offered Loads for the Links from Ground Station-to-Ground Station.

STEP E: The Erlang loss formula was expressed in vector notation in eqn. (4.2). To compute the link blocking probabilities, the information regarding the offered load and the number of trunks in the link must be known.

In our case, the information regarding the desired link[4] blocking probabilities and the load offered to the various links was obtained in steps C and D, respectively. With this information, the branch capacity vector can be obtained using equation (4.11). The number of fully variable trunks, in addition to the preassigned trunks, required can then be easily computed.

Thus, at the completion of step E we have both a routing table and a branch capacity vector. There is, however, a problem associated with the design algorithm that we have enunciated: The blocking probability of ground station-to-ground station traffic will be less than the prescribed BPMAX value, but there is a possibility that the NNGOS associated with the nodes in the terrestrial section may exceed the desired upper bound. For example, consider the augmented route tree of Fig. 4.10. The calls from node A are spilled to the ground station B. According to our algorithm, the trunks are allocated such that the NNGOS of the calls from B to C is less than the BPMAX value. The blocking probability of the calls from A to C is:

$$NNGOS_{A\to C} = 1 - x(1 - NNGOS_{B\to C})$$

According to the BPMAX requirement, $NNGOS_{A\to C} \leq BPMAX$. Then,

$$1 - x(1 - NNGOS_{B\to C})$$
$$1 - x(1 - NNGOS_{B\to C} \doteq BPMAX$$

[4]These are the links between a pair of ground stations.

But $BP_{B \to C} \doteq$ BPMAX, and therefore

$$x \doteq 1$$

if the BPMAX criterion is to be satisfied by the $NNGOS_{A \to C}$. In general, x will be less than 1 because of the finite blocking probability of the

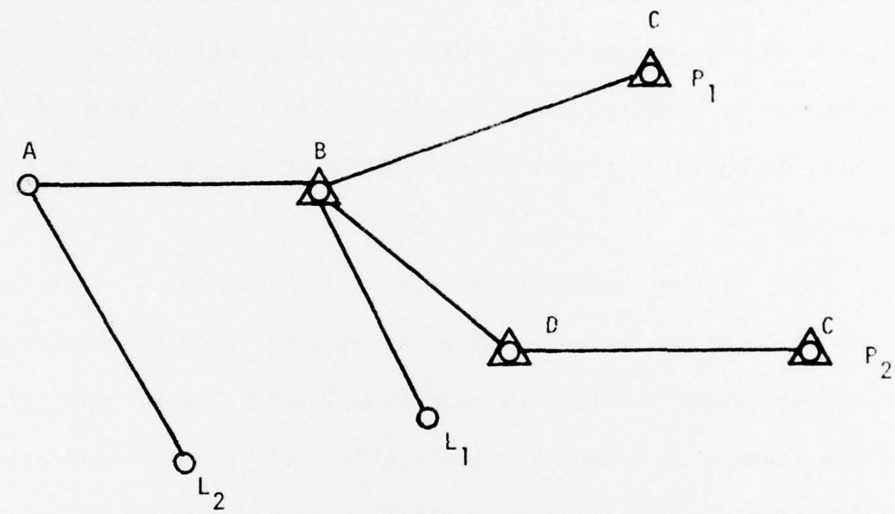

Figure 4.10 An Augment Route Tree

links between the switching center and the ground station. The NNGOS of a node pair using several tandem switches to route its calls may also exceed the desired BPMAX. Therefore, at the end of step E, the need to modify the branch capacity vector and routing table is evident. The modifications are carried out in step F.

However, before describing step F, we will illustrate step E by considering the network of Fig. 4.8. At the end of step D, the routing table and estimated offered loads to links 4 through 13 were obtained, and are shown in Table 4.6. From the offered loads and the desired link blocking probabilities (=0.103), we can obtain the required number of trunks in each link using eqn. (4.11). These results are shown in Table 4.7.

Table 4.7 Trunk Group Information after step E.

Link Number	Terminal Nodes	Number of Trunks
1	1-3	Fixed (35)
2	2-4	Fixed (28)
3	2-5	Fixed (25)
4	3-4	25
5	3-7	13
6	3-5	17
7	4-5	16
8	5-7	14
9	6-7	15
10	5-6	17
11	4-7	13
12	4-6	10
13	3-6	16

The analysis based upon the routing table given in Table 4.6 and the trunk group information in Table 4.7 is given in Fig. 4.11. We note the following salient features from the analysis:

(1) The link blocking probabilities of links 4 through 13 are not 0.103 but close to it. This is to be expected, since the capacity of a link must be an integer value.

(2) The NNGOS of the GS-to-GS nodes are less than or close to the prescribed BPMAX value.

(3) Some of the NNGOS do not satisfy the BPMAX requirement.

To improve the highest NNGOS in the network, we modify the branch capacity vector and the routing table in the next step.

<u>STEP F:</u> Before making any modifications to the routing table and/or the branch capacity vector obtained in steps D and E, it must be ascertained if such modifications would indeed improve the NNGOS that exceed the

ACTUAL TRUNK GROUP PERFORMANCE (BASED ON ORIGINAL ROUTING TABLE)

LINK NUMBER	OFFERED LOAD(CCS)	TRUNK GROUP ACTUAL GOS	TRUNK GROUP RELIABILITY
1	760.8	.001523	.998477
2	572.9	.001800	.998200
3	317.7	.000004	.999996
4	783.1	.079574	.920426
5	369.4	.092874	.907126
6	528.9	.104623	.895377
7	469.9	.087504	.912496
8	418.6	.105020	.894980
9	448.7	.099314	.900686
10	511.5	.091656	.908344
11	408.4	.130786	.869214
12	245.7	.071819	.928181
13	499.1	.110471	.889529

NETWORK PERFORMANCE (BASED ON ORIGINAL ROUTING TABLE)

SOURCE	DESTINATION	BLOCKING PROBABILITY	WEIGHTED BLOCKING PROBABILITY
1	2	8.263E-02	9.263E-02*
1	3	1.523E-03	1.523E-03
1	4	1.833E-02	1.833E-02
1	5	2.118E-02	2.118E-02*
1	6	2.170E-02	2.170E-02*
1	7	2.006E-02	2.00[illegible]E-02*
2	1	8.263E-02	8.263E-02*
2	3	1.860E-02	1.860E-02
2	4	1.800E-03	1.800E-03
2	5	4.153E-06	4.153E-06
2	6	1.403E-02	1.403E-02
2	7	2.321E-02	2.321E-02*
3	1	1.523E-03	1.523E-03
3	2	8.123E-02	8.123E-02*
3	4	1.683E-02	1.683E-02
3	5	1.968E-02	1.968E-02
3	6	2.021E-02	2.021E-02*
3	7	1.857E-02	1.857E-02
4	1	8.098E-02	8.098E-02*
4	2	1.800E-03	1.800E-03
4	3	1.683E-02	1.683E-02
4	5	1.373E-02	1.373E-02
4	6	1.559E-02	1.559E-02
4	7	2.145E-02	2.145E-02*
5	1	1.060E-01	1.060E-01*
5	2	4.153E-06	4.153E-06
5	3	1.968E-02	1.968E-02
5	4	1.373E-02	1.373E-02
5	6	1.403E-02	1.403E-02
5	7	2.172E-02	2.172E-02*
6	1	1.118E-01	1.118E-01*
6	2	9.166E-02	9.166E-02*
6	3	2.021E-02	2.021E-02*
6	4	1.559E-02	1.559E-02
6	5	1.403E-02	1.403E-02
6	7	1.919E-02	1.919E-02
7	1	9.425E-02	9.425E-02*
7	2	1.324E-01	1.324E-01*
7	3	1.857E-02	1.857E-02
7	4	2.159E-02	2.159E-02*
7	5	2.172E-02	2.172E-02*
7	6	1.918E-02	1.918E-02

```
NODE GRADE OF SERVICE (BASED ON ORIGNAL ROUTING TABLE)

NODE NUMBER        ORIGINATING LOAD (CCS)     NODE GRADE OF SERVICE        NODE RELIABILITY
     1                      453.6                  .039096                     .960904
     2                      433.6                  .018266                     .981734
     3                      831.6                  .023512                     .976488
     4                      630.0                  .022107                     .977893
     5                      756.0                  .027660                     .972340
     6                      630.0                  .036998                     .963002
     7                      730.8                  .030827                     .969173

LARGEST WEIGHTED NAGOS(= 1.324E-01) IS FROM  7 TO  2

MAXIMUM ALLOWABLE BLOCKING PROBABILITY(BPMAX)    .020

NUMBER OF WEIGHTED NAGOS EXCEEDING BPMAX IN THE ORIGNAL ROUTING TABLE= 19

NETWORK GRADE OF SERVICE IS     .02814
WEIGHTED NETWORK GRADE OF SERVICE IS     .02814
```

Figure 4.11. Performance of the Network of Fig. 4.8 at the end of Step E.

prescribed BPMAX. For example, the value of x in Fig. 4.10 could be much less than 1 so that, even if the blocking probability of calls from B to C is made zero, the blocking probability of calls from A to C is still quite high ($\approx 1-x$). In such cases, it is futile to make any modification.

But, once the decision to make the modifications is made, a criterion must be established to make such changes. To arrive at such a criterion, we note that the number of trunks in a link must have an integer value. It is therefore possible that the reliabilities of the links between the ground stations may be higher than what was actually required. We can insert or delete choice levels from the routing table to improve the highest NNGOS in the network [3]. It is possible that a few such modifications may lower all the NNGOS below the specified BPMAX value.

In the event that the above strategy fails, the next choice is to increase the capacities of the links appearing on the paths of the node pairs with the highest NNGOS. Obviously, these links must be present between two ground stations, so that fully variable trunks can be allocated to increase their traffic handling capacity.

To illustrate this step, the branch capacity vector and the routing table that satisfies the BPMAX requirement (we recall that the BPMAX requirement was not satisfied at the end of step E) is given in Table 4.8.

Table 4.8

FROM \ TO	1	2	3	4	5	6	7
1		3*	3	3*	3*	3*	3*
2	4,5		4*	4	5	5*	4*
3	1	4,5		4,7	5,7	6,7	7,4
4	3,6	2	3,7		5,6	6,7	7,6
5	3,6	2	3,7	4,6		6,4	7,4
6	3,7	5,4	3,7	4,7	5,4		7,4
7	3,6	4,5	3,4	4,3	5,4	6,3	

(a) Final Routing Table

Link Number	Terminal Nodes	Number of Trunks
1	1-3	35
2	2-4	28
3	2-5	25
4	3-4	25
5	3-7	13
6*	3-5	18
7	4-5	16
8*	5-7	15
9	6-7	15
10	5-6	17
11*	4-7	14
12	4-6	10
13*	3-6	17

(b) Final Trunk Capacity Vector

The links whose capacities were increased are indicated by an asterisk. Also, alterations in the routing table were made to improve the NNGOS.

An analysis of the network based on the information in Table 4.8 is shown in Fig. 4.12. We note that all the NNGOS are below 0.02; thus, we have completed our objective of designing the network of Fig. 4.8 such that the BPMAX criterion is satisfied.

To conclude this section, we will summarize the algorithm for trunk assignment and routing table generation:

The algorithm begins by assigning choice levels in the blocks of routing table associated with the switching centers accessing the satellite facility. The traffic spilled to the ground stations is recorded, and an augmented traffic matrix, which depicts the new traffic between ground stations, is formed.

Based on the prescribed BPMAX value and the average traffic (obtained from the augmented traffic matrix) between ground stations, the decision whether to use direct or alternate routing is made. This choice is based on which routing strategy uses fewer number of fully variable trunks. The blocking probabilities of the links between ground stations required to satisfy the BPMAX specification (for the particular routing strategy chosen) are noted.

The routing table is now constructed. If alternate routing is used, the alternate routes are generated such that the variation in the offered loads to the links between ground stations is minimal. The offered loads to these links are also recorded.

Based on the required link blocking probabilities, and their offered loads, the number of trunks needed in each link is computed using the Erlang's loss formula. An estimate of the network performance is made to check if the BPMAX criterion is satisfied. Further modifications to the link capacity vector and the routing table are made if some of the node-to-node grades of service exceed the prescribed BPMAX value.

ACTUAL TRUNK GROUP PERFORMANCE (BASED ON ORIGINAL ROUTING TABLE)

LINK NUMBER	OFFERED LOAD(CCS)	TRUNK GROUP ACTUAL GOS	TRUNK GROUP RELIABILITY
1	7[illegible]7.5	.002863	.997137
2	585.5	.002332	.997668
3	341.0	.000013	.999987
4	777.7	.076862	.923138
5	359.9	.084225	.915775
6	550.5	.093430	.906570
7	462.8	.082037	.917963
8	419.1	.075595	.924405
9	444.0	.095461	.904539
10	517.7	.096247	.903753
11	396.7	.085654	.914346
12	254.3	.081274	.918726
13	514.1	.093482	.906518

NETWORK PERFORMANCE (BASED ON ORIGINAL ROUTING TABLE)

SOURCE	DESTINATION	BLOCKING PROBABILITY	WEIGHTED BLOCKING PROBABILITY
1	2	1.023E-02	1.023E-02
1	3	2.863E-03	2.863E-03
1	4	1.533E-02	1.533E-02
1	5	1.716E-02	1.716E-02
1	6	1.886E-02	1.886E-02
1	7	1.596E-02	1.596E-02
2	1	1.023E-02	1.023E-02
2	3	1.481E-02	1.481E-02
2	4	2.332E-03	2.332E-03
2	5	1.282E-05	1.282E-05
2	6	1.509E-02	1.509E-02
2	7	1.677E-02	1.677E-02
3	1	2.863E-03	2.863E-03
3	2	7.383E-03	7.383E-03
3	4	1.250E-02	1.250E-02
3	5	1.434E-02	1.434E-02
3	6	1.605E-02	1.605E-02
3	7	1.313E-02	1.313E-02
4	1	1.567E-02	1.567E-02
4	2	2.332E-03	2.332E-03
4	3	1.250E-02	1.250E-02
4	5	1.392E-02	1.392E-02
4	6	1.406E-02	1.406E-02
4	7	1.447E-02	1.447E-02
5	1	1.970E-02	1.970E-02
5	2	1.282E-05	1.282E-05
5	3	1.434E-02	1.434E-02
5	4	1.392E-02	1.392E-02
5	6	1.508E-02	1.508E-02
5	7	1.215E-02	1.215E-02
6	1	1.886E-02	1.886E-02
6	2	8.030E-03	8.030E-03
6	3	1.605E-02	1.605E-02
6	4	1.406E-02	1.406E-02
6	5	1.508E-02	1.508E-02
6	7	1.527E-02	1.527E-02
7	1	1.798E-02	1.798E-02
7	2	6.637E-03	6.637E-03
7	3	1.313E-02	1.313E-02
7	4	1.324E-02	1.324E-02
7	5	1.215E-02	1.215E-02
7	6	1.621E-02	1.621E-02

NODE GRADE OF SERVICE (BASED ON ORIGNAL ROUTING TABLE)

NODE NUMBER	ORIGINATING LOAD (CCS)	NODE GRADE OF SERVICE	NODE RELIABILITY
1	453.6	.013112	.986888
2	453.6	.008577	.991423
3	831.6	.012937	.987063
4	630.0	.013033	.986967
5	756.0	.013971	.986029
6	630.0	.014329	.985671
7	730.8	.013449	.986551

LARGEST WEIGHTED NNGOS(= 1.970E-02) IS FROM 5 TO 1

MAXIMUM ALLOWABLE BLOCKING PROBABILITY(BPMAX) .020

NETWORK GRADE OF SERVICE IS .01298
WEIGHTED NETWORK GRADE OF SERVICE IS .01298

Figure 4.12. Performance of the Network of Fig. 4.8 at the end of Step F.

5. FURTHER RESEARCH

In section 4, we presented a heuristic algorithm for generating a branch capacity vector and a routing table such that all the node-to-node grades of service in the hybrid network are below a prescribed value. Furthermore, the number of fully variable trunks allocated to achieve the network performance criterion is minimal. To assess the usefulness of the algorithm a digital computer program should be developed. Then studies on some of the hybrid networks in planning stages can be made. Such studies will show if this algorithm is adaptable in near real-time computing[1].

The algorithm, at present, makes a tacit assumption that the traffic demands are completely different everytime this algorithm is used. In the actual operating system, this assumption may not be necessary; that is, the rate of call arrivals may change at only a few switching centers. In such cases, it is unnecessary to start the algorithm from step A; only minor alterations in the branch capacity vector and routing table may suffice. The criteria for making such alterations needs to be investigated. With a slight modification, this algorithm could be made to detect what changes in the traffic matrix have occured, and what action needs to be taken. Such modifications will undoubtedly reduce the computation time involved in certain cases.

The assumption that the network is in statistical equilibrium renders the single-moment method inapplicalbe in real-time computation, where new traffic demands are made every minute or so. Consequently,

---

[1] Based on the experience gained from developing STARTUP [3], it is envisaged that this algorithm will be useful in near real-time computing.

some other methods must be sought, which can then be applied to the routing problem in hybrid communication networks for optimum assignment on almost every demand for service.

Previous investigation [3] into the routing problem in circuit switched networks with originating office control allowed for at most one spill switch between a source-to-destination pair. Since STARTUP [3] was used as an analysis tool for the research reported in this report, the same assumption was tacitly made. With the availability of more general reliability analysis algorithms [13, 14], study into the feasibility of two or more spill switches between node pairs should be made. Such an investigation would reveal the advantages and disadvantages of "multiple" spill forward action in circuit-switched networks. Multiple spill forward action, if found advantageous, could then be used in the terrestrial section of the hybrid network.

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METRIC SYSTEM

BASE UNITS:

Quantity	Unit	SI Symbol	Formula
length	metre	m	...
mass	kilogram	kg	...
time	second	s	...
electric current	ampere	A	...
thermodynamic temperature	kelvin	K	...
amount of substance	mole	mol	...
luminous intensity	candela	cd	...

SUPPLEMENTARY UNITS:

Quantity	Unit	SI Symbol	Formula
plane angle	radian	rad	...
solid angle	steradian	sr	...

DERIVED UNITS:

Quantity	Unit	SI Symbol	Formula
Acceleration	metre per second squared	...	m/s
activity (of a radioactive source)	disintegration per second	...	(disintegration)/s
angular acceleration	radian per second squared	...	rad/s
angular velocity	radian per second	...	rad/s
area	square metre	...	m
density	kilogram per cubic metre	...	kg/m
electric capacitance	farad	F	A·s/V
electrical conductance	siemens	S	A/V
electric field strength	volt per metre	...	V/m
electric inductance	henry	H	V·s/A
electric potential difference	volt	V	W/A
electric resistance	ohm		V/A
electromotive force	volt	V	W/A
energy	joule	J	N·m
entropy	joule per kelvin	...	J/K
force	newton	N	kg·m/s
frequency	hertz	Hz	(cycle)/s
illuminance	lux	lx	lm/m
luminance	candela per square metre	...	cd/m
luminous flux	lumen	lm	cd·sr
magnetic field strength	ampere per metre	...	A/m
magnetic flux	weber	Wb	V·s
magnetic flux density	tesla	T	Wb/m
magnetomotive force	ampere	A	...
power	watt	W	J/s
pressure	pascal	Pa	N/m
quantity of electricity	coulomb	C	A·s
quantity of heat	joule	J	N·m
radiant intensity	watt per steradian	...	W/sr
specific heat	joule per kilogram-kelvin	...	J/kg·K
stress	pascal	Pa	N/m
thermal conductivity	watt per metre-kelvin	...	W/m·K
velocity	metre per second	...	m/s
viscosity, dynamic	pascal-second	...	Pa·s
viscosity, kinematic	square metre per second	...	m/s
voltage	volt	V	W/A
volume	cubic metre	...	m
wavenumber	reciprocal metre	...	(wave)/m
work	joule	J	N·m

SI PREFIXES:

Multiplication Factors	Prefix	SI Symbol
1 000 000 000 000 = 10^{12}	tera	T
1 000 000 000 = 10^{9}	giga	G
1 000 000 = 10^{6}	mega	M
1 000 = 10^{3}	kilo	k
100 = 10^{2}	hecto*	h
10 = 10^{1}	deka*	da
0.1 = 10^{-1}	deci*	d
0.01 = 10^{-2}	centi*	c
0.001 = 10^{-3}	milli	m
0.000 001 = 10^{-6}	micro	μ
0.000 000 001 = 10^{-9}	nano	n
0.000 000 000 001 = 10^{-12}	pico	p
0.000 000 000 000 001 = 10^{-15}	femto	f
0.000 000 000 000 000 001 = 10^{-18}	atto	a

* To be avoided where possible